Counting Principle & Permutations

A license plate consists of 3 letters, followed by 3 digits.

How many different license plates are possible?

$26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 17{,}576{,}000$

Suppose letters and digits may not be repeated. How many are possible?

$26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8 = 11{,}232{,}000$

Permutations

n each way

In how many ways can these letters be arranged?

$3 \cdot 2 \cdot 1$

6 ways

A B C
A C B
C A B
C B A
B A C
B C A

Permutations

In how many ways can these letters be arranged? A B C D E F G

$7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$= 7! =$ "7 factorial" $= 5040$

How many ways could I choose just two letters from the list?

$7 \cdot 6 = 42$ ways

KEY CONCEPT *For Your Notebook*

Permutations of n Objects Taken r at a Time

The number of permutations of r objects taken from a group of n distinct objects is denoted by ${}_nP_r$ and is given by this formula:

$${}_nP_r = \frac{n!}{(n-r)!}$$

$\frac{7!}{(7-2)!} = \frac{7!}{5!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

Find the number of permutations.

1. ${}_5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 60$

2. ${}_4P_1 = \frac{4!}{(4-1)!} = \frac{4!}{3!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = 4$

3. ${}_8P_5 = \frac{8!}{(8-5)!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = 6720$

Counting Principle & Permutations (cont.)

KEY CONCEPT *For Your Notebook*

Permutations with Repetition

The number of distinguishable permutations of n objects where one object is repeated s_1 times, another is repeated s_2 times, and so on, is:

$$\frac{n!}{s_1! \cdot s_2! \cdot \ldots \cdot s_k!}$$

Example: Find the number of distinguishable permutations of the letters in

SOCCER $= \frac{6!}{2!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot \cancel{2} \cdot \cancel{1}}{\cancel{2} \cdot \cancel{1}} = 360$

SWIMMING $= \frac{8!}{2!\,2!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot \cancel{4} \cdot 3 \cdot 2 \cdot 1}{\cancel{2} \cdot 1 \cdot \cancel{2} \cdot 1} = 10{,}080$

MISSISSIPPI $= \frac{11!}{4!4!2!} = \frac{11 \cdot 10 \cdot 9 \cdot \cancel{8} \cdot 7 \cdot \cancel{6} \cdot 5 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1} \cdot 2 \cdot 1} = 34{,}650$